VARIATIONAL METHODS FOR ELLIPTICAL BOUNDARY-VALUE PROBLEMS WITH NONSYMMETRIC TENSOR COEFFICIENTS

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Two perfectly different methods are examined as "variational" in [1, 2]. In the first, it is the analog of the Dirichlet principle for the Laplace equation setting up the equivalence of the solution of the boundary-value problem to the minimization of the energy functional, otherwise called energetic, while the second is the analog of the Lindeloff principle setting up the dependence of qualitative characteristics of the solution on changes in the domain shape. In principle, the methods of proving the principles are also different; however, the variational principles of both kinds are obtained most simply and completely for solutions of the Laplace equation. The principles are extended to elliptical systems of equations with variable tensor coefficients that should certainly be symmetric and positive-definite. In physical terminology, this means exclusion of gyrotropic media, conductors with Hall conductivity (plasmas or semiconductors in a magnetic field), for example, from consideration. These media are traditionally excluded also from the formulation of thermodynamic principles [3].

The author proposed a symmetric formulation of problems for gyrotropic media that permitted setting up energetic principles for fundamental and mixed two- and three-dimensional boundary-value problems [4-7].

One of the variational principles of the second kind is also extended to gyrotropic media in this paper. The proofs are elementary, but unfortunately limited to two-dimensional problems with constant coefficients. Although this particular case corresponds to the Laplace equation in a traditional formulation, a skew derivative is given on part of the boundary in the mixed boundary-value problem we are interested in. The operator of such a boundary-value problem is nonsymmetric; consequently, the variational principles were not formulated.

Formulation of the Initial Problem

Let a conducting body occupy a domain Ω. The electrical current density $\mathbf{j}$ is related by Ohm's law to the electrical field intensity $\mathbf{E}$ by the conductivity tensor $\hat{\sigma}$:

$$\frac{1}{\sigma_0}\mathbf{j} = \frac{1}{\sigma_0}\hat{\sigma}\mathbf{E} \tag{1}$$

(σ_0 is a constant whose value is dealt with below). We will use Cartesian coordinates, the components of the matrix $\hat{\sigma}$ are given function of the coordinates, and $\hat{\sigma}^+$ is the transposed matrix. The positivity of the energy liberation during passage of the electric current assures positive-definiteness of the symmetric part of $\hat{\sigma}$. Without intending to examine ideal conductors or ideal insulators inside Ω, we assume that $(\hat{\sigma} + \hat{\sigma}^+/2)$ is positive-definite and that the coefficients of $\hat{\sigma}$ are finite inside Ω.

The sharp conservation law and Maxwell equation form a system of equations in the domain Ω:

$$(1/\sigma_0)\operatorname{div}\mathbf{j} = 0, \operatorname{rot}\mathbf{E} = 0. \tag{2}$$

Let the boundary Γ of the domain Ω consist of alternating ideal conductors and insulators. In the two-dimensional case the problem with two insulators Γ_1 and Γ_3 and two conductors Γ_2 and Γ_4 is most interesting. The appropriate boundary conditions are

$$\frac{1}{\sigma_0} j_n|_{\Gamma_1,\Gamma_3} = 0,\ E_\tau|_{\Gamma_2,\Gamma_4} = 0, \tag{3}$$

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where n and τ are vector components normal and tangential to the boundary. We assume the lengths of all four sections Γ_1, Γ_2, Γ_3, and Γ_4 to be nonzero.

We consider the external electrical loop connecting the conductors Γ_2 and Γ_4 in two particular cases. Let an electrical voltage be given between Γ_2 and Γ_4 (along the insulator Γ_1 as is essential if the electrical field will be vortical)

$$-\int_{\Gamma_1} dl E_\tau = U \tag{4}$$

or a total current through the conductor Γ_4

$$-\int_{\Gamma_4} dl j_n/\sigma_0 = I/\sigma_0. \tag{5}$$

As a whole, R = U/I will be the electrical resistance of the conducting body. One of the quantities U or I is considered given below while the other is understood to be the notation for the integrals (4) or (5).

The Problem in Symmetric Form

It is customary to reduce the problem formulated above to a boundary-value problem for one second-order elliptic equation by introducing an electrostatic potential or current function. The nonsymmetry of $\hat{\sigma}$ makes the operator of such a boundary-value problem also nonsymmetric. The lack of operator symmetry does not permit application of variational methods which makes investigation as well as an approximate and numerical solution of the problems difficult. A symmetric formulation is proposed in [5] for the two-dimensional mixed boundary-value problem.

A set whose elements are pairs of smooth functions Φ, Ψ, satisfying the boundary conditions

$$\Phi|_{\Gamma_2} = 0,\ \ \Phi|_{\Gamma_4} = 0,\ \ \Psi|_{\Gamma_1} = \Psi_0,\ \ \Psi|_{\Gamma_3} = 0 \tag{6}$$

is considered (Ψ_0 is an arbitrary number).

An energetic scalar product of the elements

$$[(u, v), (\Phi, \Psi)] = \iint_\Omega dx\, dy \begin{pmatrix} \operatorname{grad} u \\ \operatorname{rot} v \end{pmatrix}^+ \begin{pmatrix} \frac{\hat{\sigma}}{\sigma_0} \hat{S} \frac{\hat{\sigma}^+}{\sigma_0} & -\frac{\hat{\sigma}}{\sigma_0}\hat{S} \\ -\hat{S}\frac{\hat{\sigma}^+}{\sigma_0} & \hat{S} \end{pmatrix} \begin{pmatrix} \operatorname{grad} \Phi \\ \operatorname{rot} \Psi \end{pmatrix}, \tag{7}$$

is introduced, where $\hat{S}$ is an arbitrary symmetric, positive-definite matrix uniform in Ω, and $\operatorname{rot}\Psi$ is a vector with the components $(\partial\Psi/\partial y, -\partial\Psi/\partial x)$.

The bilinear form (7) is a scalar product since it is symmetric and positive-definite. This latter is easily proved if it is noted that in the smooth functions satisfying conditions (6)

$$\iint_\Omega dx\, dy\, (\operatorname{grad}\Phi)^+ \operatorname{rot}\Psi = 0,$$

it is possible, by adding this integral to (7), to change the integrand quadratic form outside the principal diagonal of the block of matrices and thereby make this matrix degenerate, and uniform in the domain Ω, in addition. But this means that

$$[(\Phi, \Psi), (\Phi, \Psi)] \geqslant \text{const} \iint_\Omega dx\, dy\, ((\operatorname{grad}\Phi)^2 + (\operatorname{rot}\Psi)^2).$$

Since the function Φ equals zero on a section of the boundary, the Friedrichs inequality is valid for it:

$$\iint_\Omega dx\,dy\,(\mathrm{grad}\,\Phi)^2 \geqslant \mathrm{const} \iint_\Omega dx\,dy\,\Phi^2,$$

which is ordinarily proved for functions equal to zero on the whole boundary; however, this requirement is not necessary [8, (9.13)].

Taking into account that $(\mathrm{rot}\,\Psi)^2 = \Psi_x{}^2 + \Psi_y{}^2 = (\mathrm{grad}\,\Psi)^2$ in the two-dimensional case, an analogous inequality is valid for the function Ψ. An inequality denoting positive-definiteness $[(\Phi, \Psi), (\Phi, \Psi)] \geqslant \mathrm{const} \iint_\Omega dx\,dy\,(\Phi^2 + \Psi^2)$, is obtained, where the value of const depends on the geometry of Γ_1-Γ_4 and is independent of specific Φ, Ψ.

The matrix $\hat{S}$ and constant σ_0 should be selected so as to diminish, if possible, the ratio of the maximal (and maximal in Ω) eigennumber of such a transformed matrix to the minimal. The best estimates were successfully obtained in [4] for

$$\hat{S} = \left(\frac{\hat{\sigma} + \hat{\sigma}^+}{2\sigma_0}\right)^{-1}, \quad \sigma_0 = \sqrt{\min_\Omega \lambda \max_\Omega (\det(\hat{\sigma})/\lambda)} \tag{8}$$

[λ is the least of the eigennumbers of the symmetric matrix $(\hat{\sigma} + \hat{\sigma}^+)/2$].

The difference of the min and max from zero and infinity here is indeed the specific form of the conditions on the matrix $\hat{\sigma}$ while smoothness of the coefficients of $\hat{\sigma}$ is required in addition to the traditional approach.

The named energetic functional is considered:

$$W(\Phi, \Psi) = [(\Phi, \Psi), (\Phi, \Psi)] - 2U\Psi_0. \tag{9}$$

Since its quadratic part is positive-definite while the linear is bounded, the values of $W(\Phi, \Psi)$ have a lower bound and a minimizing sequence exists that converges to itself in the energetic norm.

In the case of smoothness of the functions Φ, Ψ that give the energy functional the minimal value, the minimality conditions for $W(\Phi, \Psi)$ agree with the original problem (2)-(4) if we use the notation

$$\mathbf{j}/\sigma_0 = -\frac{\hat{\sigma}}{\sigma_0}\hat{S}\frac{\hat{\sigma}^+}{\sigma_0}\,\mathrm{grad}\,\Phi + \frac{\hat{\sigma}}{\sigma_0}\hat{S}\,\mathrm{rot}\,\Psi,$$

$$\mathbf{E} = -\hat{S}\frac{\hat{\sigma}^+}{\sigma_0}\,\mathrm{grad}\,\Phi + \hat{S}\,\mathrm{rot}\,\Psi,$$

and the Ohm's law (1) is satisfied automatically..

In the general case we obtain a generalized solution of the problem in the sense of validity of the identity

$$\iint_\Omega dx\,dy\,(-(\mathrm{grad}\,u)^+\mathbf{j}/\sigma_0 + (\mathrm{rot}\,v)^+\,\mathbf{E}) - Uv_0 = 0$$

for the arbitrary smooth functions u, v satisfying conditions (6) (the arbitrary number v_0 is the boundary value of the function v on Γ_3). A generalized solution exists, is unique, and possesses finite energy.

The energy can be written in terms of the original problem

$$[(\Phi, \Psi), (\Phi, \Psi)] = \frac{1}{\sigma_0}\iint_\Omega dx\,dy\,\mathbf{j}^+\cdot\mathbf{E},$$

if matrix $\hat{S}$ is selected in conformity with (8). This integral is the total Joulean dissipation. If we select $\hat{S} = T((\hat{\sigma} + \hat{\sigma}^+)/2\sigma_0)^{-1}$, where the function T is the absolute temperature of the medium, then the entropy production becomes the energy in the domain under consideration

$$[(\Phi,\Psi),(\Phi,\Psi)] = \frac{1}{\sigma_0}\iint_\Omega dx\,dy\,\frac{1}{T}\,\mathbf{j}^+\cdot\mathbf{E}.$$

An arbitrary symmetric tensor $\hat\Theta$ ($\hat S = ((\hat\Theta\hat\sigma + \hat\sigma^+\Theta)/(2\sigma_0))^{-1}$) can indeed be selected as a quantity inverse to the temperature under the condition that $\hat S$ will be positive definite uniformly in Ω. Then

$$[(\Phi,\Psi),(\Phi,\Psi)] = \frac{1}{\sigma_0}\iint_\Omega dx\,dy\,\mathbf{E}^+\hat\Theta\mathbf{j}.$$

Therefore, the constructed generalized solution has meaning even from the viewpoint of thermodynamics. The multiplicity of the formulations associated with the arbitrariness of the selection of $\hat S$ corresponds to the different temperature distributions in the medium.

In the case of a current I given in the outer loop (5), the boundary conditions extracting the set of functions (Φ, Ψ) on which the energy functional should be minimized

$$\Phi|_{\Gamma_2} = 0,\quad \Phi|_{\Gamma_4} = \Phi_0,\quad \Psi|_{\Gamma_1} = 0,\quad \Psi|_{\Gamma_3} = 0 \tag{10}$$

(Φ_0 is an arbitrary number) change instead of the voltage U (4), and the linear term $W(\Phi, \Psi) = [(\Phi, \Psi), (\Phi, \Psi)] - 2I\Phi_0$ changes in the energy functional itself.

Estimate of the Resistance of the Conducting Body

To obtain the estimate we transform the energy functional. Each pair of functions (Φ', Ψ') from set (6) can be transformed into $(\Phi', \Psi') = \Psi_0(\Phi, \Psi)$, where ($\Phi$, Ψ) satisfy the conditions already fixed:

$$\Phi|_{\Gamma_2} = 0,\quad \Phi|_{\Gamma_4} = 0,\quad \Psi|_{\Gamma_1} = 1,\quad \Psi|_{\Gamma_3} = 0. \tag{11}$$

Only such normalized functions are utilized below. The energy functional (9) acquires the form $W(\Phi, \Psi) = \Psi_0{}^2[(\Phi, \Psi), (\Phi, \Psi)] - 2\Psi_0 U$. It is easily minimized in Ψ_0 for each specific pair of functions (Φ, Ψ):

$$0 = \frac{\partial W}{\partial \Psi_0} = 2\Psi_0\,[(\Phi,\Psi),(\Phi,\Psi)] - 2U,$$

i.e.,

$$\Psi_0 = U/[(\Phi,\Psi),(\Phi,\Psi)],\ W = -U^2/[(\Phi,\Psi),(\Phi,\Psi)]. \tag{12}$$

Since U is a given number, the minimality of W corresponds to minimality of the energy (Φ, Ψ).

By virtue of the energy conservation law, the total dissipation in a conducting body equals the energy influx from the outer loop, i.e., for the exact solution $\Psi_0{}^2[(\Phi, \Psi), (\Phi, \Psi)] = (1/\sigma_0)UI$.

Together with (12) this yields the expression of the exact value of Ψ_0 in terms of the current I, unknown in advance: $\Psi_0 = I/\sigma_0$. Now the resistance can be expressed in just the terms of the minimal energy (Φ, Ψ):

$$R = \frac{U}{I} = \min\frac{1}{\sigma_0}\,[(\Phi,\Psi),(\Phi,\Psi)]. \tag{13}$$

Since functions giving the minimal value to the energy figure here, the energy of any other (Φ, Ψ) satisfying (11) yields the upper bound for R. Analogously, the energy of any functions satisfying the simplified ($\Phi_0 = 1$) condition (10) yields the lower bound of the estimate for R.

Expressions of the form (13) and the corresponding estimates are also valid for the resistance of three-dimensional bodies. However, the specific form of the energy is complicated substantially: the function $\mathbf{\Psi}$ becomes a vector corresponding to the vector equation rot $\mathbf{E}$ = 0(2); the square of the divergence $\mathbf{\Psi}$, is appended to the energy, which is associated in the long run with overdefiniteness of the original three-dimensional system of equations (1) and (2). Assignment of the boundary conditions for the vector function $\mathbf{\Psi}$ even requires preliminary construction of a harmonic function on the section of the boundary corresponding to the insulator (this function is determined by the geometry of the boundary and yields a method for averaging the electrical field in the insulator during calculation of the voltage between the conductors, which was done in a trivial manner in the two-dimensional case). The problem is examined in detail in [7], and the three-dimensional problems with a homogeneous boundary (the whole boundary is an insulator or a superconductor) in [6].

Resistance of Two-Dimensional Homogeneous Conductors

For isotropic media with the scalar conductivity $\sigma_\parallel$ in a magnetic field perpendicular to the x, y plane, the following kind of conductivity tensor is characteristic:

$$\hat{\sigma} = \frac{\sigma_\parallel}{1+\beta^2}\begin{pmatrix} 1 & -\beta \\ \beta & 1 \end{pmatrix}$$

(β is the Hall parameter proportional to the magnetic-field intensity). We examine below precisely such $\hat{\sigma}$ since for constant coefficients the problem is reduced to this form simply by rotations and stretching of the coordinate system.

For homogeneous media when $\sigma_\parallel$ and β are constants, the resistance of the conductor as a whole is expressed more simply:

$$R = \min \iint_\Omega dx\, dy \{ \sqrt{1+\beta^2} (\Phi_x^2 + \Phi_y^2 + \Psi_x^2 + \Psi_y^2) - - 2\beta (\Phi_x \Psi_x + \Phi_y \Psi_y) \} \sqrt{1+\beta^2}/\sigma_\parallel. \tag{14}$$

According to (8),

$$\sigma_0 = \sigma_\parallel / \sqrt{1+\beta^2}, \quad \hat{S} = \sqrt{1+\beta^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

were fixed here.

The term $(\Phi_x\Psi_t - \Phi_y\Psi_x)$ is omitted at once since its integral equals zero by virtue of the boundary conditions (11). Minimization of the energy (14) by functions satisfying the boundary conditions (11) is equivalent, in this case, to solving the boundary-value problem

$$\Delta\Phi = 0, \quad \Delta\Psi = 0, \quad \Phi|_{\Gamma_2,\Gamma_4} = 0, \quad \Psi|_{\Gamma_1} = 1, \quad \Psi|_{\Gamma_3} = 0,$$
$$-\sqrt{1+\beta^2}\frac{\partial\Phi}{\partial n} + \beta \frac{\partial\Psi}{\partial n}\Big|_{\Gamma_1,\Gamma_3} = 0, \quad \beta\frac{\partial\Phi}{\partial n} - \sqrt{1+\beta^2}\frac{\partial\Psi}{\partial n}\Big|_{\Gamma_2,\Gamma_4} = 0. \tag{15}$$

Conformal Mapping into a Rectangle

For β = 0 the functional and the boundary problem (15) for Φ and Ψ split. We obtain $\Phi = 0$ and $\Delta\Psi = 0$, $\Psi|_{\Gamma_1} = 1$, $\Psi|_{\Gamma_3} = 0$, $\partial\Psi/\partial n|_{\Gamma_2,\Gamma_4} = 0$.

The energy is transformed into the conformal capacitance

$$C = \min \iint dx\, dy\, (\Psi_x^2 + \Psi_y^2) \tag{16}$$

and

$$R = C/\sigma_\parallel. \tag{17}$$

The existence of a mutually one-to-one conformal mapping of the curvilinear quadrangle under consideration into a rectangle with correspondence of the four angles is utilized in the proof presented below. A mapping with correspondence of three angles exists as for every simply connected univalent domain with piecewise-smooth Jordan boundaries [9]. Since the capacitance (16) is an invariant of conformal mappings, while the capacitance of a rectangle is x_0/y_0, we take

$$x_0 = 1, \; y_0 = 1/C. \tag{18}$$

Then it is easily proved that the fourth angle also goes over into an angle of the rectangle.

We now expand the domain Ω in such a manner that a new section of the boundary Γ_1^* lies outside Ω. We predefine the function Ψ obtained during minimization of the energy (13) in Ω by $\Psi = 1$ in the added part of the domain. The energy is not changed here. The energy does not grow during minimization in the new domain Ω^*, which means $C^* \leq C$. Now, if we map Ω^* into a rectangle, as was described above, we obtain $y_0^* \geq y_0$ from (18).

Resistance of Homogeneous Gyrotropic Rectangles

For homogeneous rectangles the problem (15) acquires a still more specific form

$$\Delta\Phi = 0,\ \ \Delta\Psi = 0,\ \ \Phi|_{x=0,1} = 0,\ \ \Psi|_{y=0} = 1,\ \ \Psi|_{y=y_0} = 0,$$
$$-\sqrt{1+\beta^2}\,\Phi_y + \beta\Psi_y|_{y=0,y_0} = 0,\ \ \beta\Phi_x - \sqrt{1+\beta^2}\,\Psi_x|_{x=0,1} = 0.$$

If the number 1/2 is subracted from Ψ, we obtain a problem antisymmetric with respect to the line $y = y_0/2$. Correspondingly, its solution is also antisymmetric, meaning

$$\Phi|_{y=y_0/2} = 0,\ \ \Psi|_{y=y_0/2} = 1/2. \tag{19}$$

To analyze the change in resistance during expansion of the rectangle from y_0 to $y_0^* > y_0$, we divide the initial rectangle into halves by the line $y = y_0/2$ and insert a strip of width $y_0^* - y_0$ in the slit by predefining $\Phi = 0$, $\Psi = 1/2$ in it. By virtue of (19), the functions Φ and Ψ remain continuous in the whole new domain and the value of the energy does not change. The energy is not increased during minimization. Consequently, the resistance R which is proportional to the minimal energy (14) does not increase during expansion of the rectangle.

Formulation of the Variational Principle. Let us combine three assertions: expansion of a rectangle into which it is mapped conformally with correspondence of the four angles, corresponds to expansion of a curvilinear rectangle; the energy and the resistance (14) there by during conformal mappings do not change, and are confirmed directly; the resistance of a rectangle does not decrease during expansion. We obtain the variational principle: during expansion of a conductor its resistance does not grow.

As was remarked above, upper bounds are found for the resistance during minimization of the energy of functions satisfying opposite boundary conditions [(10) instead of (6)]. An increase in the domain outside the conductors Γ_2 or Γ_4 is conveniently examined in this formulation. In the two-dimensional case such a problem differs only by notation from that presented; consequently, the second part of the principle is also valid: during elongation of a conductor its resistance does not decrease. The position of the angles does not change in both cases.

Conclusions

In addition to the methodological value in principle, the principle obtained has a practical application since it permits replacement of the domain during estimation of the resistance of Hall conductors by a simpler one. In particular, it is necessary to perform triangulation for a variational-difference realization of the upper bound such that the broken lines approximating the boundary with the insulator would pass within the domain while the approximating boundary to the ideal conductor would pass outside the domain. In this and only in this case, is the value obtained numerically is the true upper bound. For the lower bound the broken lines must be constructed in an opposite manner. The efficiency of the proposed symmetrization for the numerical solution of the problems is demonstrated in [10, 11].

The boundedness of the proofs does not permit asserting the validity of the formulated principle for variable coefficients; however, in the two-dimensional cases foundations for doubts are not seen.

Let us note that the proof of the energetic principles in [4-7] is carried out for a sufficiently broad circle of problems. Piecewise continuity, uniform boundedness, and uniform positive-definiteness of its symmetric part were required from the conductivity tensor $\hat{\sigma}$.

LITERATURE CITED

1. S. G. Mikhlin, Variational Methods in Mathematical Physics [in Russian], Nauka, Moscow (1970).
2. M. A. Lavrent'ev, Variational Methods in Boundary-Value Problems for Systems of Equations of Elliptic Type [in Russian], Izd. Akad. Nauk SSSR, Moscow (1962).
3. D'yarmati, Nonequilibrium Thermodynamics. Theory of Fields and Variational Principles [Russian translation], Mir, Moscow (1974).
4. V. V. Denisenko, "Symmetric form of elliptic equations describing transport processes in gyrotropic media," Deposited paper No. 4896-80, VINITI, November 5, 1980, Krasnoyarsk (1980).
5. V. V. Denisenko, "Boundary-value problem for a symmetric strongly elliptic system under a changing type of boundary condition," Deposited paper No. 1008-81, VINITI, March 4, 1981, Krasnoyarsk (1981).
6. V. V. Denisenko, "Symmetric form of three-dimensional elliptic equations describing transport processes in gyrotropic media," Deposited paper No. 344-81, VINITI, January 28, 1981, Krasnoyarsk (1981).
7. V. V. Denisenko, "Energetic method of solving three-dimensional mixed boundary-value problems for an elliptic equation with nonsymmetric coefficient matrix," Deposited paper No. 5647-81, VINITI, December 14, 1981, Krasnoyarsk (1981).
8. S. L. Sobolev, Certain Applications of Functional Analysis in Mathematical Physics [in Russian], Izd. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1962).
9. A. V. Bitsadze, Principles of the Theory of Analytic Complex Functions [in Russian], Nauka, Moscow (1972).
10. V. V. Denisenko and S. S. Zamai, "Symmetrization and numerical solution of elliptic boundary-value problems with nonself-adjoint operators," in: Modeling of Hydrogasdynamics and Energetics Problems [in Russian], Inst. Teor. Prikl. Mekh., Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1984).
11. V. V. Denisenko and S. S. Zamai, "Numerical modeling of electrical fields and currents in a low-latitude ionosphere," in: Mathematical Models and Methods of Solution of Problems of Mechanics of a Continuous Medium [in Russian], Vychisl. Tsentr, Sib. Otd. Akad. Nauk SSSR, Krasnoyarsk (1986).

STABILITY OF THE SURFACE OF A GAS BUBBLE PULSATING IN A LIQUID

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This article examines the stability of the surface of a spherical gas bubble undergoing nonlinear oscillations. We study the dynamics of small perturbations as a function of the wavelength and parameters of the nonlinear bubble pulsations. An approach is developed for analyzing the dynamics of the bubble-surface perturbations on the basis of solution of the differential equation of stability for a pulsation half-period. The shortwave approximation is used to obtain a formula for the increment of the perturbation, and an analogy is established between the stability problem and the problem of the passage of a particle across a potential barrier in quantum mechanics. Asymptotic formulas are found for the rate of growth of perturbations in the case of large-amplitude pulsations, and a comparison is made with exact numerical calculations. It is shown that the rate of growth of perturbations of a prescribed wavelength is a bounded function with infinite intensification of the pulsations. With consideration of capillary forces, it was found that the most rapidly growing perturbations shift in the shortwave direction as the amplitude of the pulsations intensifies. It is shown that Taylor instability is the main reason for rupture of the surface of the pulsating gas bubble.

The stability of a plane interface between two liquids was first examined by Taylor [1] in connection with the problem of bubble dynamics in an underwater explosion. Experiments

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